Multi-resonance orbital model of high-frequency QPOs: possible high precision determination of black hole and neutron star spin

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Aims and scope

Using known frequencies of the twin peak quasiperiodic oscillations (QPOs) and the known mass of the central black hole, the black-hole dimensionless spin a can be determined, assuming a concrete version of the resonance model. However, because of large range of observationally limited values of the black hole mass, its spin can be estimated with a low precision only. The LOFT project enables a large improvement in the precision of the timing measurements that can have some important consequences. We discuss the possibility of higher precision of the black hole dimensionless spin measurements in the framework of multi-resonance models of QPOs inspired by observations of more than two resonant frequencies in some sources. We determine the spin and mass dependence of the twin peak frequencies for non-linear resonances of oscillations with the epicyclic and Keplerian frequencies or their combinations in the case of a general rational frequency ratio $n : m$, $n > m$. In the multi-resonant model, the twin peak resonances are combined properly to give the observed frequency set. We focus on the special case of duplex frequencies, when the top, bottom, or mixed frequency is common for the two different radii where the resonances occur giving triple frequency sets.

Orbital resonance models involving Keplerian and epicyclic oscillations

(A) More instances of one or more resonances occurring at specific radii

This kind of the multi-resonance model is probably relevant in the neutron star binary systems, where data clustering of the twin peak QPOs is observed for the ratios $3:2$, $4:3$, $5:4$ in six atoll sources [1, 2].

(B) More resonances sharing one specific radius

This special case allows existence of strong resonant phenomena, since the Keplerian and both epicyclic frequencies are in the rational ratios in a shared radius and cooperative phenomena between different kinds of resonance could appear. Of course, such a situation is allowed for black holes with a specific spin only. The very important triple frequency set with ratios $\nu_K:\nu_\theta:\nu_r = 3:2:1$ is then given by the case of the "magic" spin $a = 0.983$, when the resonances share the radius $r/M = x = 2.395$ [3] (see Figure 1). In this magic case, the combinational frequencies give the same frequency ratios. Therefore, the combinational and the simple frequency oscillations could be in the $1:1$ ratio, corresponding to the strongest possible resonant phenomena. In such a case, the scatter of the resonant frequencies could be the highest one, indicating a possible implication to the frequency set (probably) observed in Sgr A* [4]. The mass range is then in agreement with limits given by other observations.

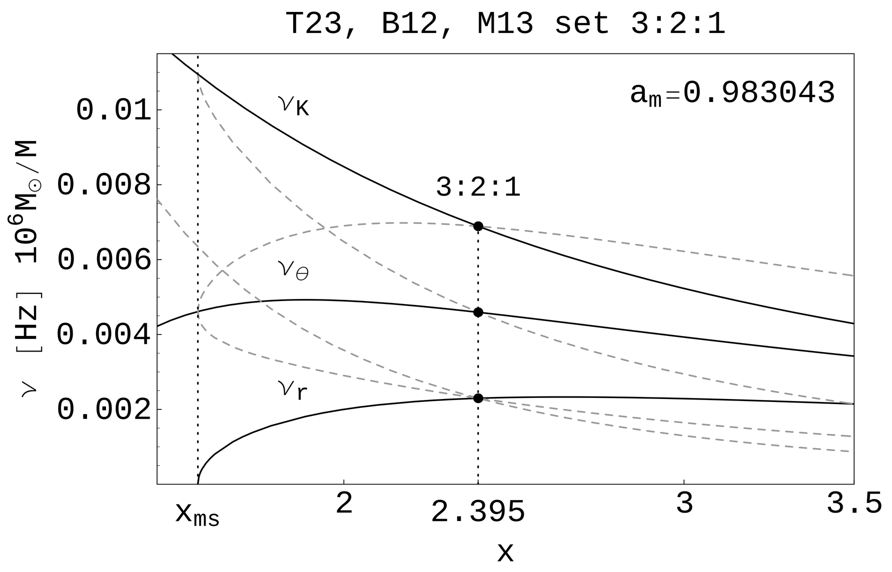

Fig. 1: The case of a "magic" spin, when the strongest resonances could occur at the same radius. For completeness we present the relevant simple combinational frequencies $\nu_\theta - \nu_r$, $\nu_\theta + \nu_r$, $\nu_K - \nu_\theta$, $\nu_K - \nu_r$ (grey dashed lines). Notice that the "magic" spin $a_m = 0.983$ represents the only case when the combinational and direct orbital frequencies coincide at the shared resonance radius.

(C) More resonances occurring at more specific radii (the "ugly" case)

In general, we can expect the oscillations to be excited at two (or more) different radii of the accretion disc that enter the forced or parametric resonance in the framework of different versions of the resonance model. In such situations we are making the black hole spin estimate within two properly chosen versions of the resonance model, obtaining thus in principle more precise determinations of the spin than in the case, when only one twin peak is observed. In special cases, when a common upper (lower, mixed) frequency is observed in the two frequency pairs (i.e., only three different frequencies are observed), the triple frequency set is precisely given for specifically fixed values of the black hole spin independently of the black hole mass. Then, the spin is in principle determined precisely (within the precision of the frequency measurements), but not uniquely, as in general the same frequency set could occur for different values of the spin within different versions of the resonance model. It is clear that in such situations the black hole spin estimates coming from the spectra fitting and the line profile model could be relevant in determining the proper versions of the resonant model. When the black hole spin is found, its mass can be determined from the magnitude of the observed frequencies.

Results

The typical cases of the frequency triple sets with bottom, top and both types of mixed identities containing duplex frequencies are illustrated in Figure 2.

In [5] we present a detailed Table guide across all the possible triple frequency sets and related values of the black hole spin a, shown for all possible double combinations of both the direct and simple combinational resonances with the order of individual resonances limited by $n \leq 5$. The resonances are considered up to $n = 5$ as the excitation of higher order resonances is highly improbable.

Discussion and conclusions

The multi-resonance model of high-frequency QPOs can be considered as an interesting and promising approach to understand the observational data from LMXB containing both black holes and neutron stars. The special case of the triple frequency set method determines the black hole spin with very high precision and quite independently of the measurement of the black hole mass, but it could work only incidentally, for special values of the spin. However, it is worth to make a detailed scan of all the observational data for the LMXB black hole systems or supermassive black holes in active galactic nuclei in order to look for some candidate systems, since any success in precise determination of the spin could help very much in determining other physical parameters of the system and to understand a wide scale of astrophysical phenomena.

The efficiency of the black hole spin determination by using the triple frequency set ratios grows strongly with growing precision of the frequency measurements. Therefore, in the case of measurements of very high precision, the method could work even for the frequency sets ratio of high integers, because in resonances of high order the frequencies in the resonance must be tuned very fine in order to let the resonance to work [6].

On the other hand, in the LMXB neutron star sources (both atoll and Z-sources) the situation is quite different and a significantly different multi-resonance model is probably relevant due to the wide range of the observed twin peak high-frequency QPOs that has to be approximated by one frequency relation or two similar relations predicting in the atoll sources coupling of the data near small integer frequency ratios $3:2$, $4:3$, $5:4$. Nevertheless, due to the existence of the energy switch effect [1] strongly suggesting presence of resonant phenomena even in kHz QPOs observed in the neutron star systems, the multi-resonance model can be applied there in an appropriately modified approach. Further, the neutron star spacetime parameter estimates given by the multi-resonant approach has to be in an accord with estimated coming from fitting all the observational data of twin peak QPOs with frequency relations following all the range of the data. We expect differences between the black hole and neutron star resonant phenomena to be attributed to the presence of the neutron star surface and related inhomogeneities influencing the high-frequency QPOS in vicinity of the star surface.

We can conclude that the multi-resonant model of QPOs based on the orbital motion is capable to explain a wide range of QPO phenomena observed in both black hole and neutron star X-ray binary systems, around the supermassive galactic centre (Sgr A*) black hole or some other black hole systems in the centre of active galactic nuclei, and in the intermediate (NGC 5408 X-1) black holes systems. However, some basic properties of resonant phenomena in black hole and neutron star systems differ, probably because of the phenomena related to the surface of the neutron stars. The data analysis indicates that in neutron star systems probably one frequency relation could not explain data clustering around the resonant points with ratios $3:2$, $4:3$, $5:4$ and require a non-geodesic correction or more complex approach [7]. In the black hole systems, the QPO data indicate presence of different versions of the multi-resonance model in concrete sources. We could speculate that the strong resonant phenomena allowed in black hole systems with special values of dimensionless spin could be observationally preferred because of wide range of (possibly) cooperating resonant phenomena. One of candidates for such a system seems to be the central black hole in Sgr A*. For near-extreme black holes, an extended resonance model could be relevant [8]. Clearly, a lot of observational and theoretical research is necessary for deeper understanding to the resonant phenomena indicated in the black hole systems and we hope that the multi-resonant model will turn to be very useful in the future research.

Further, we expect complementary information from profiled spectral lines generated near the inner edge of the accretion disc that could help in testing exotic hypothetical sources as Kerr superspinars [9, 10] or superspinning strange stars [11].

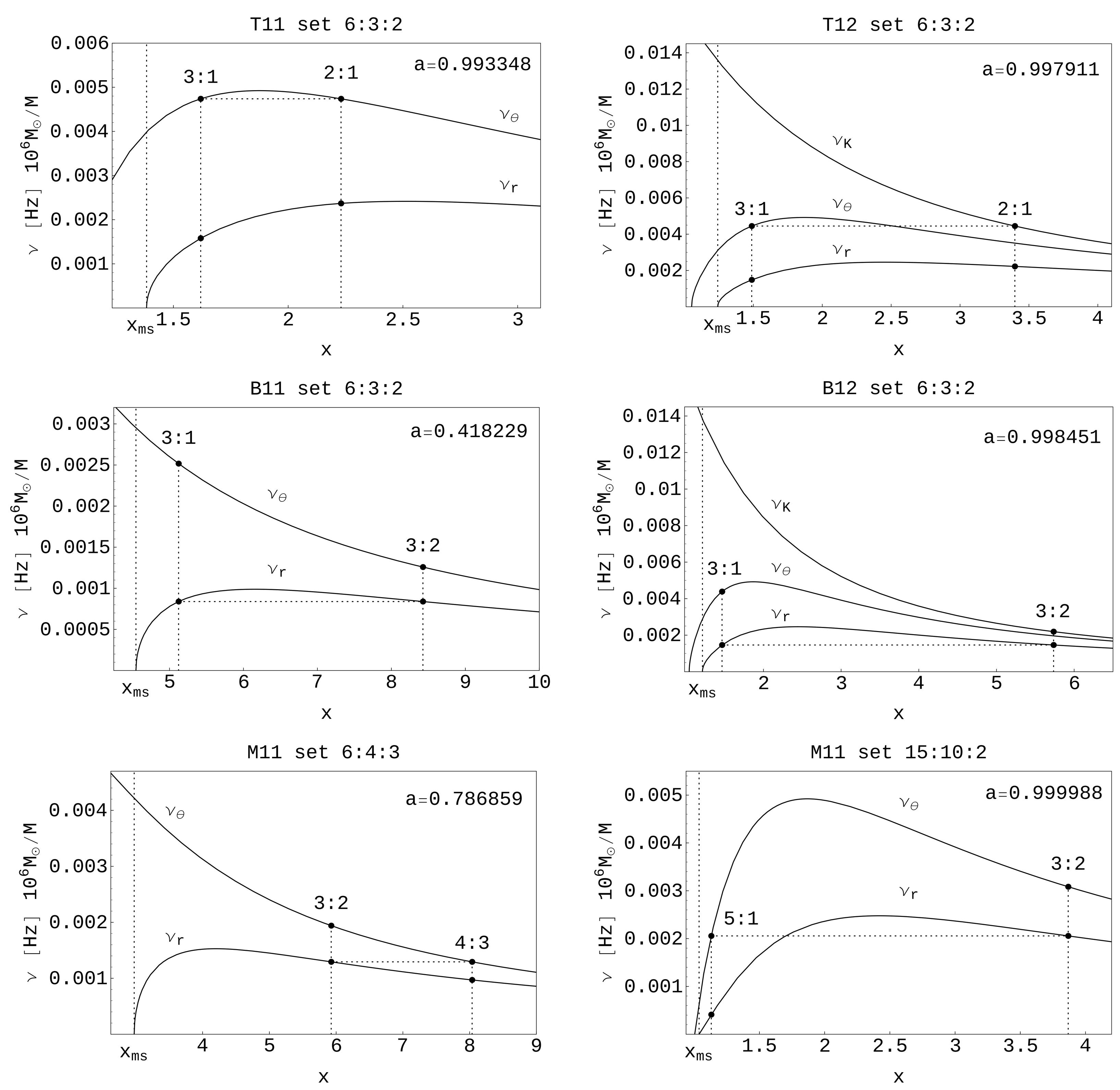

Fig. 2: The typical cases of the frequency triples with top, bottom, and both types of mixed identities.

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